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第三章 涡量动力学
Chapter 3 Vorticity Dynamics

October 29, 2024

3.1 Introduction

There are two ways to describe fluid motion:

In the ***Lagrangian*** description, **fluid particles** are followed as they move through a flow field.

In the ***Eulerian*** description, a flow field's characteristics are monitored at fixed locations or in stationary regions of space.

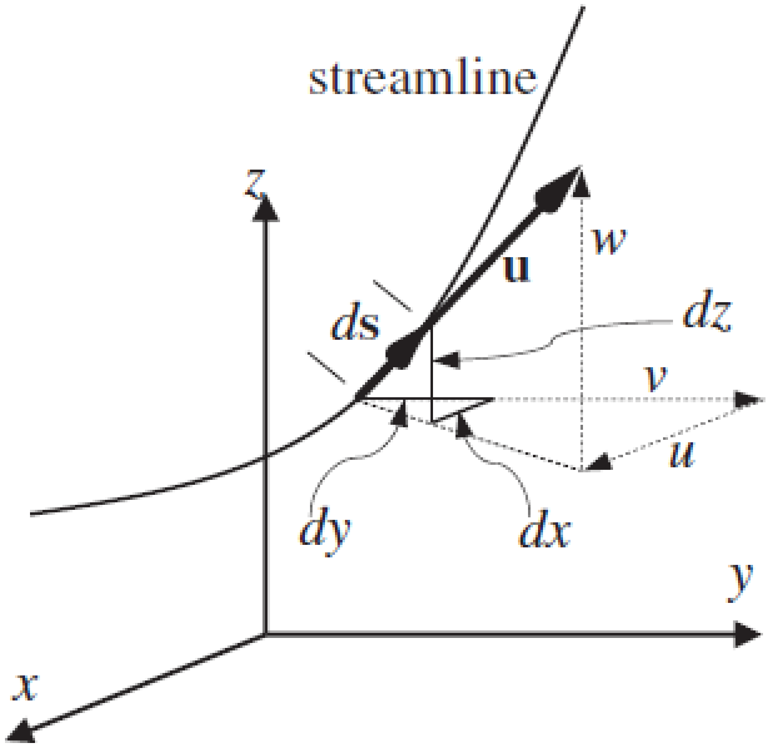
3.1 Introduction

In fluid mechanics, an understanding of both descriptions is necessary because the acceleration following a fluid particle is needed for the application of Newton's second law to fluid motion

while observations, measurements, and simulations of fluid flows are commonly made at fixed locations or in stationary spatial regions with the fluid moving past the locations or through the regions of interest.

3.1 Introduction

A ***streamline*** is a curve that is instantaneously tangent to the fluid velocity throughout the flow field.



3.1 Introduction

A ***path line*** is the trajectory of a fluid particle of fixed identity.

A ***streak line*** is the curve obtained by connecting all the fluid particles that will pass or have passed through a fixed point in space.

3.2 Vorticity

Vorticity is a vector field that is twice the angular velocity of a fluid particle.

For example, we can associate the vector $\boldsymbol{\omega}$ having components ω_i, with an antisymmetric tensor:

$$\mathbf{R} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}.$$

$$R_{ij} = \sum_{k=1}^{3} -\varepsilon_{ijk}\omega_k \equiv -\varepsilon_{ijk}\omega_k, \quad \text{and} \quad \omega_k = \sum_{i-1}^{3}\sum_{j=1}^{3} -\frac{1}{2}\varepsilon_{ijk}R_{ij} \equiv -\frac{1}{2}\varepsilon_{ijk}R_{ij}$$

3.2 Vorticity

Fluid motion leading to circular or nearly circular streamlines is called ***vortex motion***.

In two dimensions(r, θ), a uniform distribution of plane-normal vorticity with magnitude ω produces solid body rotation

$$u_\theta = \omega r/2$$

3.2 Vorticity vs vortex vs vortices

Vortex and vorticity are two correlated but fundamentally different concepts which have been the central issues in fluid mechanics research.

***Vorticity*（涡量）** has rigorous mathematical definition (curl of velocity), but no clear physical meaning.

***Vortex* （涡旋）** has clear physical meaning (rotation) but no rigorous mathematical definition.

Vortices is the plural of the word vortex.

3.2 Vorticity

The ***circulation*** Γ is defined by

$$\Gamma \equiv \oint_C \mathbf{u} \cdot d\mathbf{s} = \int_A \boldsymbol{\omega} \cdot \mathbf{n} dA$$

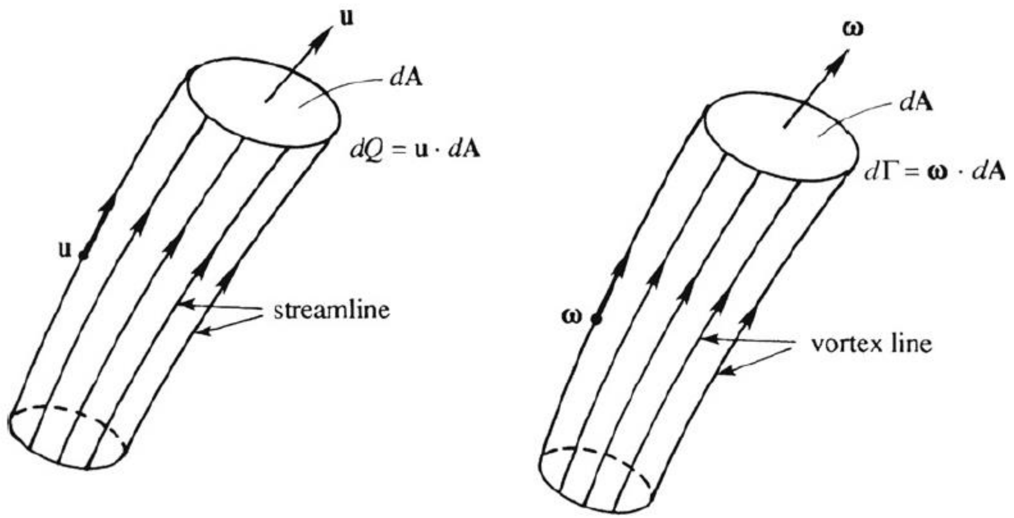
The circulation around a circuit of radius r is

$$\Gamma = \oint_C \mathbf{u} \cdot d\mathbf{s} = \int_0^{2\pi} u_\theta r d\theta = 2\pi r u_\theta = 2\pi r^2 \omega_0$$

3.2 Vorticity

A ***vortex line*** is a curve in the fluid that is everywhere tangent to the local vorticity vector.

A vortex line is related to the vorticity vector the same way a streamline is related to the velocity vector



Stream tube   Vortex tube

3.3 Kelvin's Circulation Theorem

By considering the analogy with electrodynamics, Helmholtz(赫尔姆霍兹) published several theorems for vortex motion in an inviscid fluid in 1858.

Ten years later, Kelvin introduced the idea of circulation and proved the following theorem:
In an inviscid, barotropic(正压的) flow with conservative body forces, the circulation around a closed curve moving with the fluid remains constant with time, if the motion is observed from a nonrotating frame. This theorem can be stated mathematically as

$$D\Gamma/Dt = 0$$

3.3 Kelvin's Circulation Theorem

Kelvin's theorem implies that irrotational flow will remain irrotational if the following four restrictions are satisfied.

(1) There are no net viscous forces along C.

(2) The body forces are conservative.

(3) The fluid density must depend on pressure only (barotropic flow).

(4) The frame of reference must be an inertial frame.

3.4 Helmholtz's Vortex Theorems

Under the same four restrictions, Helmholtz proved the following theorems for vortex motion:

(1) Vortex lines move with the fluid.

(2) The strength of a vortex tube (its circulation) is constant along its length.

(3) A vortex tube cannot end within the fluid. It must either end at a solid boundary or form a closed loop----a *vortex ring* or loop.

(4) The strength of a vortex tube remains constant in time.

3.5 Gallery of Fluid Motion

3.5 Gallery of Fluid Motion

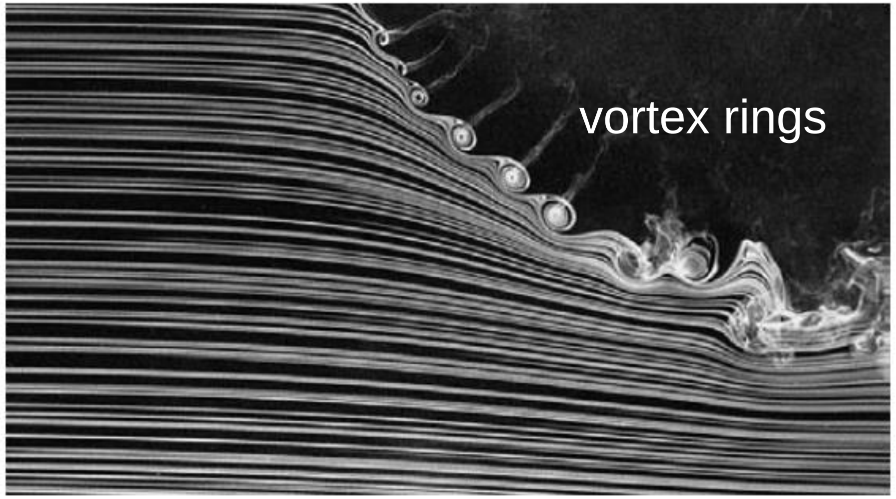


Figure 1

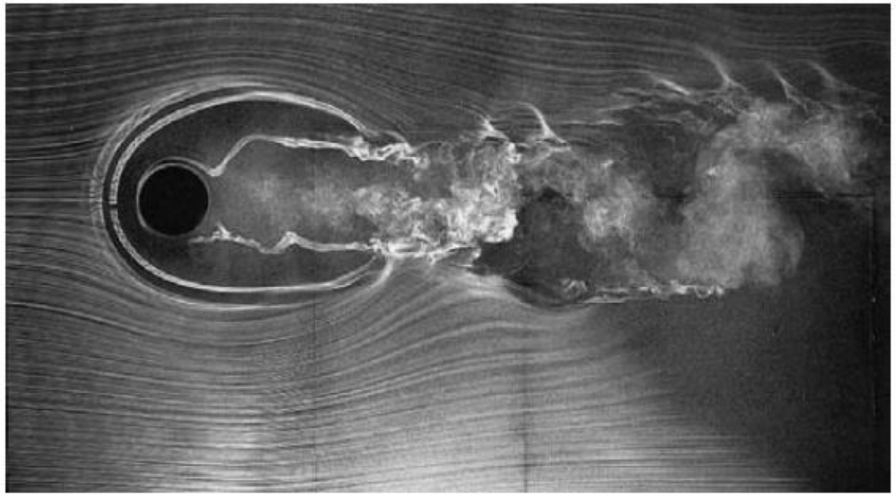


Figure 2

horseshoe
vortex system

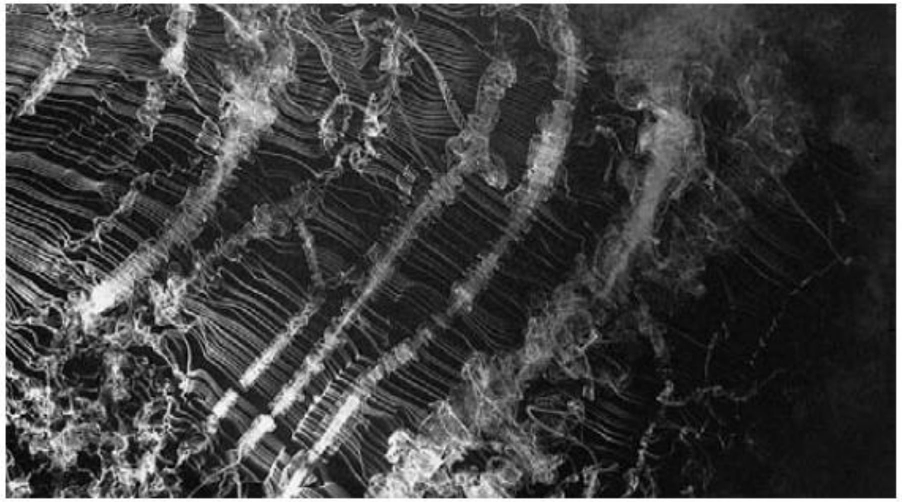


Figure 3

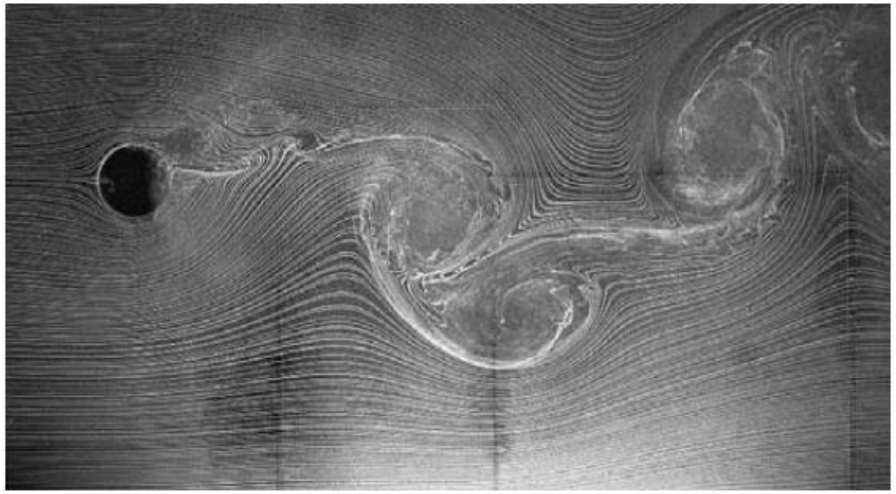


Figure 4

smoke
visualization

3.5 Gallery of Fluid Motion

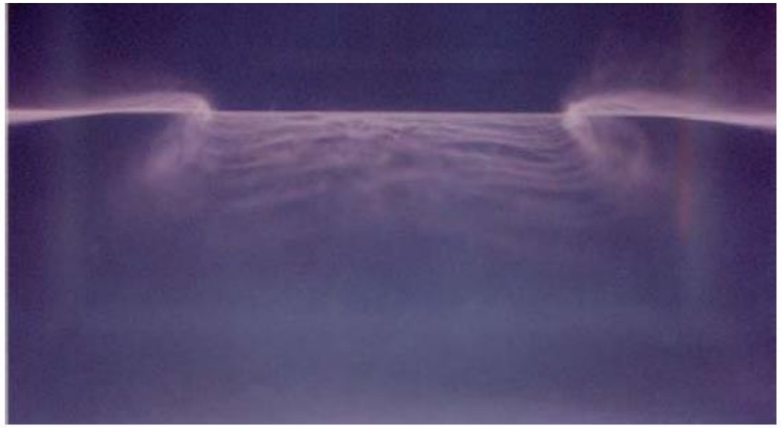


Figure 1 Model study (end view).

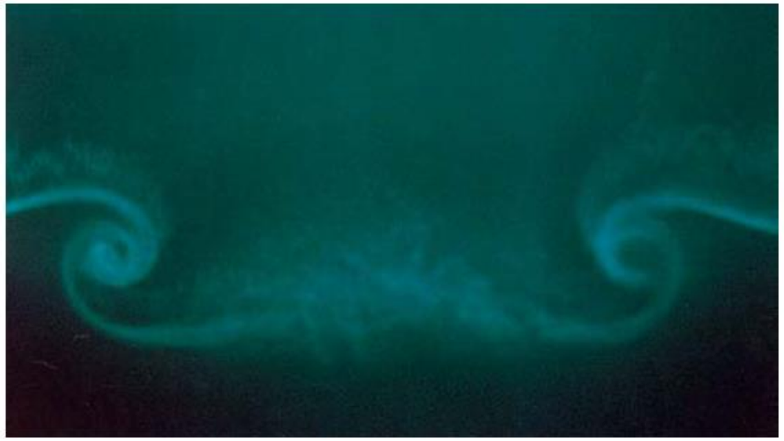


Figure 2 Model study (cross-sectional view).

Figure 3 Aircraft wake (photo courtesy of Cessna Aircraft Company).

Aircraft trailing vortices and downwash phenomenon

3.5 Gallery of Fluid Motion

The picture shows the wake of three cylinders placed in line and an equal distance apart with a ratio between the surface of the cylinders and cylinder diameter of 5.7.

3.5 Gallery of Fluid Motion

Figure 1. Circular cylinder, Re. = 100

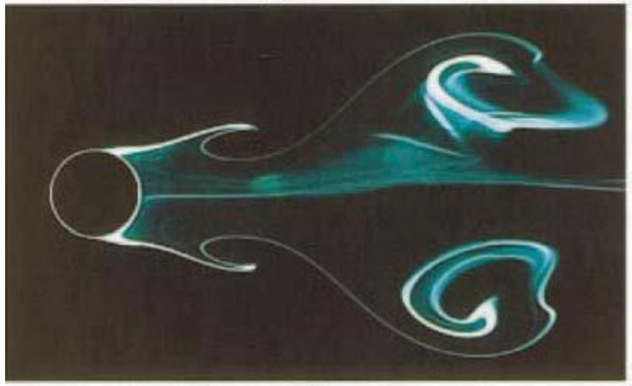
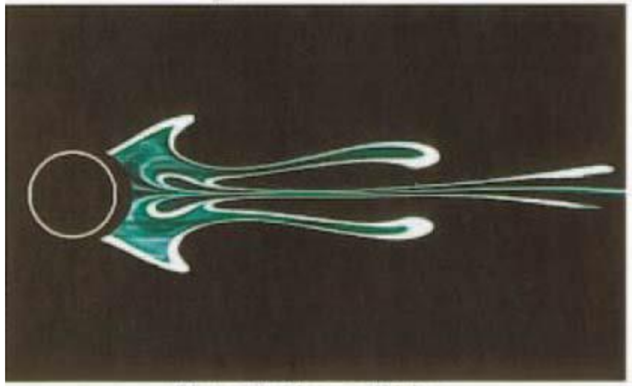
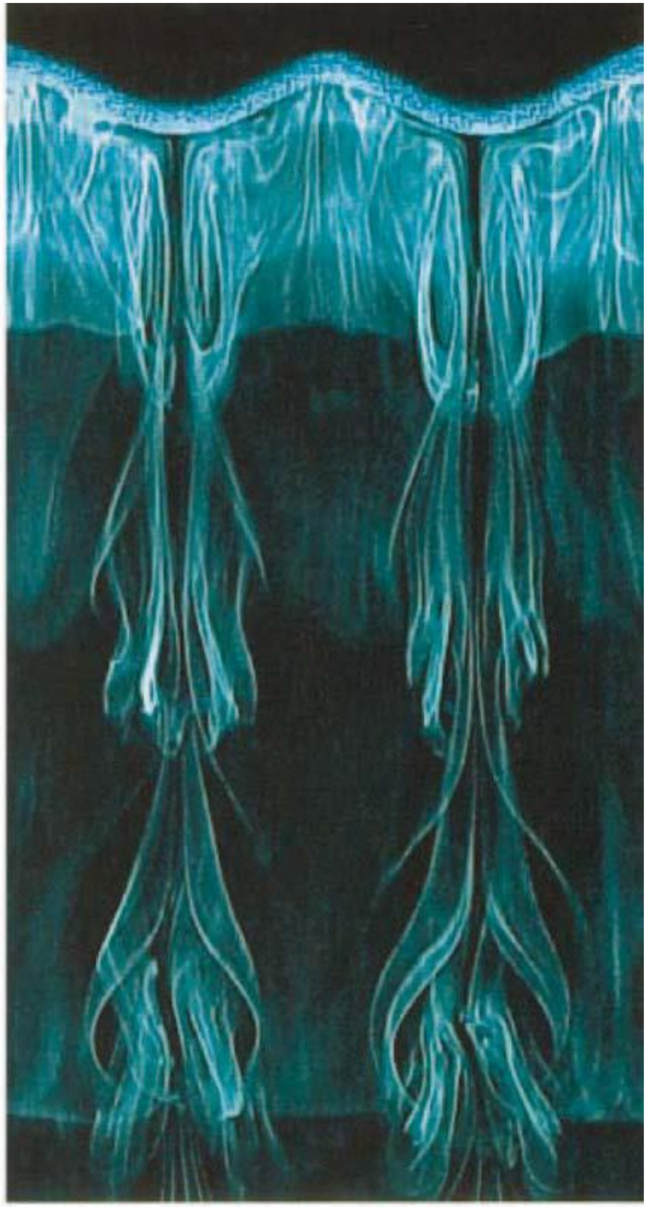


Figure 2. Wide Wake





Suppression of Kármán vortex shedding

3.5 Gallery of Fluid Motion

Air entrainment by a plunging jet translating over a free surface

The photographs above illustrate the mechanism by which a steady vertical laminar jet (D_{jet} = 6 mm, velocity V_{jet}= 303 cm/s) induces air entrainment as it translates horizontally over a quiescent pool (velocity V_t= 44 cm/s from right to left).